PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1842. No. 33.

January 24.

REV. HUMPHREY LLOYD, D. D., Vice-President, in the Chair.

RESOLVED,—On the recommendation of Council,—That it shall be the duty of the Committee of Publication to report on Papers intended for publication in the Transactions.

RESOLVED,—On the recommendation of Council,—That the Treasurer be authorized to sell stock of the Academy to the amount of £300, for payment of the Printer's bill and other arrears.

The Rev. Charles Graves, F. T. C. D., read a paper "on the Motion of a Point upon the Surface of a Sphere."

When the motion of a material point is limited to a given plane, the circumstances of its motion are commonly investigated by means of the equations,

$$\mathrm{X}dt^2 = d^2x, \quad \mathrm{Y}dt^2 = d^2y,$$

x and y being rectangular coordinates in the given plane. Mr. Graves shows that, in like manner, the motion of a material point, constrained to move on the surface of a sphere, whose radius = 1, may be discussed by means of the similar equations,

$$\text{X}dt^2 = d\left(\frac{dx}{1+x^2+y^2}\right) \quad (1) \qquad \text{Y}dt^2 = d\left(\frac{dy}{1+x^2+y^2}\right) \qquad (2)$$

in which x and y are used to denote *the rectangular spherical coordinates* of the moving point (vid. page 127), and X, Y, the moments, in the planes of the x and y arcs of reference, of the resultant of the forces acting upon the point. The reaction of the surface being taken into account, this resultant is tangential to the sphere, and so may be conceived to act along a great circle passing through the point.

From equations (1) and (2) we derive a third,

$$(\text{X}y - \text{Y}x)\,dt^2 = d\left(\frac{ydx - xdy}{1+x^2+y^2}\right) \qquad (3)$$

which leads to important consequences.

It appears from the second formula in p. 129 that, if the equations (1) (2) and (3) be multiplied respectively by

$$\frac{2\,dx}{1+x^2+y^2}, \quad \frac{2dy}{1+x^2+y^2}, \quad \text{and} \quad \frac{2(ydx-xdy)}{1+x^2+y^2},$$

they will give for the velocity, v, of the moving point,

$$2\int\frac{\text{X}\,[dx+y(ydx-xdy)] + \text{Y}\,[dy+x\,(xdy-ydx)]}{1+x^2+y^2} = v^2. \qquad (4)$$

Now, if the resultant tangential force R act always along a great circle which passes through the origin, that is, if we consider the case analogous to that of a central force in the dynamics of the plane,

$$\text{X} = \text{R}\,\frac{x}{(x^2+y^2)^{\frac{1}{2}}}, \text{ and } \text{Y} = \text{R}\,\frac{y}{(x^2+y^2)}.$$

In this case, therefore, which for simplicity we may call that of a *central* force, equation (3) gives

$$\frac{ydx - xdy}{1+x^2+y^2} = hdt, \qquad (5)$$

and equation (4) is reduced to

$$2\int \frac{\text{X}dx + \text{Y}dy}{1 + x^2 + y^2} = v^2. \tag{6}$$

It is easy to show that these two latter equations are equivalent to the two following,

$$\sin^2\rho d\omega = hdt, \tag{7}$$

$$2\int \text{R}d\rho = v^2, \tag{8}$$

in which ρ is the vector arc drawn from the origin to the moving point, and ω is the angle between it and the x arc of reference. Let us now describe the circle of the sphere which osculates the trajectory along which the point P moves, and let C be the arc of the great circle passing through P and the origin, and intercepted within the osculating circle; then it may be shown that

$$\text{R} = \frac{v^2}{\tan \frac{1}{2} \text{C}}. \tag{9}$$

If p denote the arc of the great circle drawn from the origin perpendicular to the arc touching the trajectory at P, we may deduce from (7) that

$$v^2 = \frac{h^2}{\sin^2 p}. \tag{10}$$

By the help of equation (9) it may be proved that "A material point may be made to describe a spherical conic if it be urged by a force, acting along the arc of a great circle drawn from the *focus* to the point, and varying inversely as the square of the sine of the vector arc ρ."

Also: "A material point may be made to describe a spherical conic by the agency of a force, acting along the arc of a great circle drawn from the *centre* to the point, and varying as $\tan\rho \sec^2\rho$."

In the dynamics of a point constrained to move on the surface of a sphere, we have, for the discussion of the inverse problem of *central* forces, the following equation,

$$\text{R} = h^2(1 + u^2)\left(u + \frac{d^2u}{d\omega^2}\right),$$

in which u is the cotangent of ρ.

The analogy between the formulæ given in this paper and those usually employed in discussing the motion of a point on a plane is very striking. The former too become identical with the latter when the portion of the sphere on which the trajectory is described becomes infinitely small in comparison with the radius.

The Rev. H. Lloyd V. P. read the following paper "on a New Magnetical Instrument, for the Measurement of the Inclination, and its Changes."

In order to know all that relates to the earth's magnetic force, at a given place, observation must furnish the values of three elements. Those which naturally present themselves for immediate determination are, the *intensity* of the force itself, and the two angles (the *declination* and *inclination*) which determine its direction. We may substitute for these, however, any other system of elements which are connected with them by known relations. Thus, we have hitherto preferred to observe the *declination*, and the *two components* (horizontal and vertical) *of the intensity;* and, in general, the main considerations which should guide us in our choice are, the exactness of the observed results, and the facility of their determination.

In this point of view, the *declination* and the *horizontal component of the intensity* leave us nothing to desire, their determination being now reduced to a degree of precision, hardly (if at all) inferior to that of astronomical measurements. The same thing, however, cannot be said respecting the third element, as hitherto observed. In the Dublin Magnetical Observatory, and in the Observatories since established by order of the Government and of the East India Company upon the same plan, the third element chosen for